POWER COUNTING FOR AND SYMMETRIES OF THE EFFECTIVE FIELD THEORY FOR NN INTERACTIONS

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The appropriate power counting for the effective field theory of NN interactions is reviewed. It is more subtle than in most effective field theories since in the limit that the S-wave NN scattering lengths go to infinity it is governed by a nontrivial fixed point. The leading two body terms in the effective field theory for nucleon self interactions are scale invariant and invariant under Wigner SU(4) spin-isospin symmetry in this limit. Higher body terms with no derivatives (i.e., three and four body terms) are automatically invariant under Wigner symmetry.

1   Basics

Consider $N(\mathbf{p})N(-\mathbf{p}) \to N(\mathbf{p}')N(-\mathbf{p}')$ scattering in the $^1S_0$ channel. Since the spins of the two nucleons are combined anti-symmetrically Fermi statistics implies that this channel is $I = 1$ (similarly the $^3S_1$ and $^3D_1$ channels are $I = 0$). The energy $E = p^2/M = p'^2/M$ where $p^{(')} = |\mathbf{p}^{(')}|$ and the scattering matrix, S, is related to the scattering amplitude $\mathcal{A}$ by $S = 1 + iMp\mathcal{A}/2\pi$. Since $S = e^{2i\delta}$, where δ is the phase shift,

$$\mathcal{A}^{(^1S_0)} = \frac{4\pi}{M} \frac{1}{p \cot \delta^{(^1S_0)} - ip}, \tag{1}$$

where M is the nucleon mass. For $p < m_\pi/2$ the quantity $p \cot \delta$ can be expanded in a power series in p^2

$$p \cot \delta^{(^1S_0)} = -\frac{1}{a^{(^1S_0)}} + \frac{1}{2} r_0^{(^1S_0)} p^2 + \dots, \tag{2}$$

when a is called the scattering length and r_0 is called the effective range. The scattering length in the $^1S_0$ channel is very large,[1] $a^{(^1S_0)} \simeq -23.7$ fm or $1/a^{(^1S_0)} \simeq -8.3$ MeV. On the other hand the nuclear potential is characterized by a momentum scale $\Lambda \sim 200$ MeV. The smallness of $|1/a^{(^1S_0)}|$ compared with this scale is the result of an accidental cancellation which causes a state in the spectrum to be very near zero binding energy. ($a \to -\infty$ as a scattering state approaches zero energy and $a \to \infty$ as a bound state approaches zero binding energy.) Neglecting the small $^3S_1 - ^3D_1$ mixing, formulas analogous to eqs. (1) and (2) hold in the $^3S_1$ channel. The scattering length is also large in that

case,[1] $a^{(^3S_1)} \simeq 5.4$ fm or $1/a^{(^3S_1)} \simeq 36$ MeV. The bound state in this channel that is near zero binding energy is the deuteron.

2 Expansions of $\mathcal{A}$

The simplest expansion of $\mathcal{A}$ is a momentum expansion. This is analogous to what is done in standard applications of effective field theory, e.g., chiral perturbation theory for $\pi\pi$ scattering. For NN scattering in the $s = {}^1S_0$ or $^3S_1$ channels,

$$\mathcal{A}^{(s)} = \frac{4\pi}{M} \frac{1}{\left[-1/a^{(s)} + \frac{1}{2}r_0^{(s)}p^2 + \ldots - ip\right]}$$

$$= -\frac{4\pi}{M} a^{(s)} \left\{ 1 - ia^{(s)}p + \left(\frac{a^{(s)}r_0^{(s)}}{2} - a^{(s)2}\right)p^2 + \ldots \right\}. \tag{3}$$

If $a^{(s)}$ was its natural size (i.e., $a^{(s)} \sim 1/\Lambda$) this would be the appropriate expansion to perform. However, in nature the S-wave NN scattering lengths are very large and the expansion above is only valid in the small region of momentum $p \lesssim |1/a^{(s)}| \ll 1/\Lambda$. Since the underlying physics is set by m_π and Λ_{QCD} there should be an expansion in p/Λ that is valid even when $p \gg |1/a^{(s)}|$. It is not difficult to deduce what this expansion is. In Eq. (3) keep $-1/a^{(s)} - ip$ in the denominator and expand in the remaining terms. This yields

$$\mathcal{A}^{(s)} = -\frac{4\pi}{M} \frac{1}{(1/a^{(s)} + ip)} \left[1 + \frac{r_0^{(s)}p^2/2}{(1/a^{(s)} + ip)} + \ldots \right]. \tag{4}$$

Now $\mathcal{A}^{(s)} = \sum_{n=-1}^{\infty} \mathcal{A}_n^{(s)}$, where $\mathcal{A}_n^{(s)} \sim \mathcal{O}(p^n)$. This is the appropriate expansion in the case where the scattering lengths are large. It has the unusual property that the leading term is order p^{-1}.

3 Effective Field Theory Without Pions

The effective field theory with the pions integrated out contains only nucleon fields, $N = \binom{p}{n}$, and we expect that the lowest dimension operators will be the most important ones. The Lagrange density is written as, $\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2 + \ldots$, where $\mathcal{L}_n$ contains n-body operators. The one and two body terms are:

$$\mathcal{L}_1 = N^\dagger \left[i\partial_t + \frac{\vec{\nabla}^2}{2M} \right] N + \ldots, \tag{5}$$
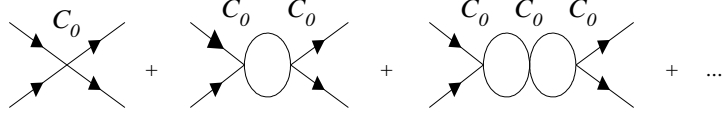
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Figure 1: The leading contribution to NN scattering.

$$\mathcal{L}_2 = -\sum_s C_0^{(s)}(N^T P_i^{(s)} N)^\dagger (N^T P_i^{(s)} N) + \dots . \tag{6}$$

Here $s = {}^1S_0$ or ${}^3S_1$, the ellipses denote higher dimension operators and $P_i^{(s)}$ are the spin-isospin projectors

$$P_i^{(^1S_0)} = \left(\frac{\sigma_2 \tau_2 \tau_i}{\sqrt{8}}\right), P_i^{(^3S_1)} = \left(\frac{\sigma_2 \sigma_i \tau_2}{\sqrt{8}}\right), \tag{7}$$

where the Pauli matrices σ_i act in spin space and the Pauli matrices τ_i act in isospin space.

Neglecting higher dimension operators the scattering amplitudes in the ${}^1S_0$ and ${}^3S_1$ channels come from the sum of bubble-type Feynman diagrams shown in Fig. 1. Each bubble is linearly divergent in the ultraviolet so the coefficients $C_0^{(s)}$ depend on the regulator and subtraction scheme adopted. We use dimensional regularization and start with minimal subtraction (we will switch to a different subtraction scheme momentarily). Since the divergences are linear the Feynman diagrams have poles at $D = 3$ but not at $D = 4$. In MS (minimal subtraction) the coefficients of the operators explicitly displayed in Eq. (6), are subtraction point independent and we denote them by $\bar{C}_0^{(s)}$. In this scheme the sum of bubble-type Feynman diagrams gives

$$\mathcal{A}^{(s)} = -\frac{\bar{C}_0^{(s)}}{1 + iMp\bar{C}_0^{(s)}/4\pi}. \tag{8}$$

Comparing Eq. (8) with eqs. (1) and (2) it is evident that this corresponds to keeping only the scattering length term in the expansion of $p \cot \delta^{(s)}$, (i.e., the first term of Eq. (4)) and that

$$\bar{C}_0^{(s)} = \frac{4\pi a^{(s)}}{M}. \tag{9}$$

So in this subtraction scheme the coefficients $\bar{C}_0^{(s)}$ are very large and also very different in the two channels. However as $a^{(s)} \to \infty, \mathcal{A}^{(s)} \to 4\pi i/Mp$ which is

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the same in both channels. This form for the scattering amplitudes is consistent with Wigner spin-isospin $SU(4)$ symmetry, and also with scale invariance.

In MS when $p > 1/a^{(s)}$ the terms in the perturbative series for the scattering amplitude get larger and larger. We would like to use a subtraction scheme where the various Feynman diagrams in Fig. 1 are the same size as their sum and where the symmetries that arise as $a^{(s)} \to \infty$ are manifest at the level of the Lagrangian. Examples of such subtraction schemes are [2] PDS where poles at $D = 3$ are also subtracted and the OS momentum space subtraction scheme. [3,4] In these schemes the coefficients are subtraction point dependent, $C_0^{(s)} = C_0^{(s)}(\mu)$, and the sum of bubble diagrams gives

$$\mathcal{A}^{(s)} = -\frac{C_0^{(s)}(\mu)}{1 + M(\mu + ip)C_0^{(s)}(\mu)/4\pi}. \tag{10}$$

This still corresponds to keeping just the scattering length, and is the leading term in Eq. (4). But now

$$C_0^{(s)}(\mu) = -\frac{4\pi}{M} \frac{1}{\mu - 1/a^{(s)}}, \tag{11}$$

which as $a^{(s)} \to \infty$ becomes $C_0^{(s)}(\mu) = -4\pi/M\mu$. In this limit, the coefficients are the same in both channels and with $\mu \sim p$ each term in sum of bubble type Feynman diagrams in Fig. 1 is the same size as the sum itself.

The operators with coefficients $C_0^{(s)}$ are nonrenormalizable dimension six operators. Naively they are irrelevant operators and at low momentum can be treated in perturbation theory. However as $a^{(s)} \to \infty$ the coefficients $C_0^{(s)}(\mu)$ flow to a nontrivial fixed point[2,3] where $\mu d[\mu C_0^{(s)}(\mu)]/d\mu = 0$. For large $a^{(s)}$ the power counting is controlled by this fixed point and the leading contribution to the NN scattering amplitude comes from treating $C_0^{(s)}$ nonperturbatively. It is straightforward to show that in PDS or OS the coefficients of S-wave operators with $2n$ spatial derivatives scale as, [2]

$$C_{2n}^{(s)}(\mu) \sim \frac{4\pi}{M\Lambda^n \mu^{n+1}}, \tag{12}$$

for $\mu \gg |1/a^{(s)}|$. With $\mu \sim p, C_{2n}^{(s)}(\mu)p^{2n} \sim p^{n-1}$ and the two body operators with derivatives can be treated perturbatively. In a non-relativistic theory a loop integration $\int d^4q = \int dq^0 d^3q \sim \mathcal{O}(p^5)$ (since the dq^0 integration is of order p^2 and the d^3q is order p^3) and the nucleon propagator $i/(p^0 - p^2/2M + i\epsilon) \sim \mathcal{O}(p^{-2})$. Consequently each loop gives a factor p plus whatever factors of p are associated with the vertices. The power counting [2,5]

is now evident. The leading order (LO) contribution $\mathcal{A}_{-1}^{(s)}$ comes from $C_0^{(s)}$ treated nonperturbatively, the next to leading order (NLO) contribution $\mathcal{A}_0^{(s)}$ comes from $C_0^{(s)}$ treated nonperturbatively and $C_2^{(s)}$ inserted once, the next-to-next to leading order (N^2LO) contribution comes from $C_0^{(s)}$ treated non-perturbatively, $C_2^{(s)}$ inserted twice or $C_4^{(s)}$ inserted once, etc.

With the pions integrated out the effective field theory expansion applied to NN scattering reproduces Eq. (4) and has no more content than the momentum expansion of $p \cot \delta^{(s)}$. However, even with the pions integrated out one can couple photons or W and Z gauge bosons to the nucleons. The relative importance of operators containing these fields depends on their renormalization group scaling near the fixed point.

In the two nucleon sector predictions based on the effective field theory without pions are similar to those made by effective range theory. [6] However the effective field theory approach has a number of advantages. Predictions based on effective range theory are only valid to a given order in the p/Λ expansion. In the effective field theory new two-body operators containing the gauge fields arise which spoil the predictions of effective range theory. For the thermal neutron capture cross section, $\sigma(n + p \to d + \gamma)$, this occurs at NLO while for the deuteron matter (charge) radius $< r_m >$ this doesn't occur until N^3LO. This explains why the effective range theory prediction for $\sigma(n + p \to d + \gamma)$ is off by 10% while the effective range theory prediction for $< r_m >$ is accurate to better than a percent.[7] For these static deuteron properties the relevant momentum in the p/Λ expansion is set by the deuteron binding energy, i.e., $p \sim 40$ MeV. Another useful aspect of the effective field theory formalism is that it is straightforward to include relativistic corrections.

As $a^{(s)} \to \infty$, $\mathcal{L}_2 \to -(2\pi/M\mu)(N^\dagger N)^2 + \ldots$, where the ellipses denote two body operators with derivatives. In this limit the leading one and two body terms are invariant under the following symmetries: [8]

(i.) Wigner Symmetry [9]

Under infinitesimal Wigner symmetry $SU(4)$ transformations

$$\delta N = i\alpha_{\mu\nu}\sigma^\mu\tau^\nu N, \tag{13}$$

where $\sigma^\mu = (1, \boldsymbol{\sigma})$ and $\tau^\mu = (1, \boldsymbol{\tau})$ with $\mu = 0, 1, 2, 3$ and repeated indices summed. The symmetry group corresponding to Eq. (13) is actually $SU(4) \times U(1)$, with α_{00} the group parameter for the additional baryon-number $U(1)$. Associated with this symmetry are the conserved charges,

$$Q^{\mu\nu} = \int d^3x N^\dagger \sigma^\mu \tau^\nu N. \tag{14}$$

The two body terms with derivatives are not invariant under Wigner symmetry even if $a^{(s)} \to \infty$. Hence in the two body sector the violations of Wigner symmetry go as, $(1/[a^{(^1S_0)}p] - 1/[a^{(^3S_1)}p])$ and p/Λ. Wigner symmetry will not be a good approximation if the momentum p is too low or if it is too large.

Wigner symmetry is relevant for nuclei with many nucleons. [10] It is not difficult to see that the higher body terms with no derivatives are automatically invariant under Wigner symmetry. Since these contact terms are antisymmetric in the nucleon fields N and in the hermitian conjugates $N^\dagger$, contact terms without derivatives cannot occur for five body operators and higher. The nucleons N are in the $\mathbf{4}$ of $SU(4)$ and the $N^\dagger$'s are in the $\bar{\mathbf{4}}$. Four nucleons combined anti-symmetrically are an $SU(4)$ singlet and so the four-body terms are invariant under $SU(4)$. The three body terms transform as $\bar{\mathbf{4}} \otimes \mathbf{4} = \mathbf{1} \otimes \mathbf{15}$. However the operators in the $\mathbf{15}$ are not invariant under the total spin or isospin $SU(2)$ subgroups of $SU(4)$. Hence the allowed three body terms are also invariant under $SU(4)$ Wigner symmetry.

A complete extension of the general fixed point power counting to the higher body terms has not been made. However there has been considerable recent progress. [5] This work indicates that the 3-body term with no derivatives is leading order (i.e., as important as effects coming from $C_0^{(s)}$).

(ii.) Scale Invariance

The leading one and two body terms are invariant under the scale transformation $N(t, \mathbf{x}) \to N'(t, \mathbf{x})$ and $\mu \to \mu'$ where

$$N'(t, \mathbf{x}) = \lambda^{-3/2} N(t/\lambda^2, \mathbf{x}/\lambda), \tag{15}$$

$$\mu' = \mu/\lambda. \tag{16}$$

Note that Eq. (15) corresponds to $N'(t', \mathbf{x}') = \lambda^{-3/2} N(t, \mathbf{x})$ with $\mathbf{x}' = \lambda \mathbf{x}$ and $t' = \lambda^2 t$. The different scaling of space and time coordinates is dictated by invariance of the leading one-body terms in the Lagrange density.

4   Including Pions

With pions included the power counting is taken to be in powers of Q/Λ_{NN} where $p \sim m_\pi \sim Q$. A subscript NN has been put on Λ as a reminder that
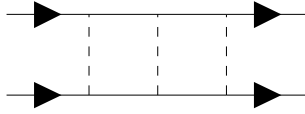
Figure 2: Contribution to $^3S_1$ scattering from three potential pion exchange. The dashed lines denote potential pion exchange.

the expansion should work better if the pions are included as explicit fields, i.e., we expect that $\Lambda_{NN} > \Lambda$. Potential pion exchange arises from the term

$$\mathcal{L}_{\text{int}} = -\frac{g_A}{\sqrt{2}f_\pi}\nabla^i\pi^j N^\dagger\sigma^i\tau^j N, \tag{17}$$

where $g_A \simeq 1.25$ is the axial coupling and $f_\pi \simeq 131$ MeV is the pion decay constant. Exchange of a potential pion between nucleons is order Q^0 (the two factors of Q from the vertices cancel the $1/Q^2$ from the pion propagator). This is the same size as the two body contact terms with two derivatives and consequently pion exchange can be treated perturbatively. Including pion exchanges without the two derivative two body contact terms is not a systematic improvement and is no better (from a power counting perspective) than just including the effects of $C_0^{(s)}$. Note that this power counting is very different from the one originally proposed by Weinberg [3,12] where the leading contribution came from treating both potential pion exchange and $C_0^{(s)}$ nonperturbatively. The effects of two body terms with derivatives and insertions of the light quark mass matrix were considered subdominant.

Weinberg's power counting treats the nucleon mass M as large and $MQ \sim \mathcal{O}(1)$. It assumes that factors of M only arise from the loop integrations. In a toy model where perturbative matching between the full relativistic theory and the nonrelativistic effective theory can be explicitly performed Luke and Manohar [13] found that the two body local operators in the effective nonrelativistic theory have coefficients that contain factors of M. This is the origin of the problem with Weinberg's power counting.

In the $^3S_1$ channel the Feynman diagram shown in Fig. 2 with three potential pion exchanges is logarithmically divergent. Neglecting the pion mass
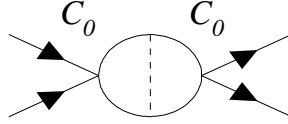
Figure 3: Contribution to NN scattering that renormalizes $D_2^{(s)}$.

it gives a contribution to $\mathcal{A}^{(^3S_1)}$ of order

$$\left(\frac{4\pi}{M}\right)\left(\frac{Mg_A^2}{8\pi f_\pi^2}\right)^3 p^2[\ln\mu^2 + K], \tag{18}$$

where K is a constant. The μ-dependence above is cancelled by the μ-dependence of $C_2^{(^3S_1)}$. There is no point to including this Feynman diagram without including the effects of the two body $^3S_1$ operator with 2-derivatives. Eq. (18) is an N^3LO contribution. With the pions included a single insertion of $C_2^{(s)}$ is not just NLO it contributes at higher levels at the Q expansion as well. For that reason $C_2^{(s)}$ and the other contact term coefficients are sometimes written as a sum $C_2^{(s)} = \sum_{a=1}^{\infty} C_{2,a}^{(s)}$ where $C_{2,1}^{(s)}$ gives the NLO contribution, etc. When this is done predictions for physical quantities are exactly μ independent, at each order in the Q expansion. If $C_2^{(s)}$ is not expanded in this way then predictions at a given order on the Q expansion have some subtraction point dependence, which is higher order in the Q expansion.

There are two body S-wave contact terms with no derivatives but with an insertion of the light quark mass matrix,

$$m_q = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix}. \tag{19}$$

Since $m_\pi^2 \propto (m_u + m_d)$ an insertion of m_q counts as two powers of Q and the coefficients of these operators $D_2^{(s)}$ scale with μ in the same way as the coefficients $C_2^{(s)}$. At NLO they must also be included. The Feynman diagram in Fig. 3 is logaritmically divergent and it gives a contribution to the $^1S_0$ and $^3S_1$ scattering amplitudes of order

$$\left(\frac{4\pi}{M}\right)\left(\frac{g_A^2 M}{8\pi f_\pi^2}\right)\left(\frac{C_0^{(s)} M}{4\pi}\right)^2 m_\pi^2[\ln\mu^2 + K], \tag{20}$$

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where K is a constant. The μ dependence here is cancelled by that of the coefficients $D_2^{(s)}$. Including one pion exchange without the effects of the two body terms with one insertion of the quark mass matrix does not systematically improve the theoretical prediction for the NN scattering amplitude.

If a momentum cutoff regulator is used instead of dimensional regularization then including pion exchange without the two body contact operators that have an insertion of the quark mass matrix results in a cutoff dependent amplitude $\mathcal{A}^{(s)}$. It is possible in the $^1S_0$ channel to sum to all orders potential pion exchange and when this is done the cutoff dependence does not become subdominant [14] (compared with the finite cut off independent parts of pion exchange). The effects of local four nucleon (i.e., two body) operators with an insertion of the quark mass matrix cannot be viewed as less important than the effects of pion exchange.

The conventional explanation for the discrepancy between the prediction of effective range theory for the thermal neutron capture cross section $\sigma(n + p \to d + \gamma)$ and its experimental value is meson exchange currents, [15] which roughly speaking are the contribution of Feynman diagrams where the photon couples to a potential pion. In the effective field theory approach with the pions included this discrepancy is made up [16] (at least partly) by the NLO contribution which involves both meson exchange current Feynman diagrams and the contribution of a local two body operator involving the magnetic field.

5 An Application of Wigner Symmetry

Potential pions have $k^0 \sim \mathbf{k}^2/M$ while radiation pions have $k^0 \sim \sqrt{\mathbf{k}^2 + m_\pi^2}$. The coupling of the radiation pions to the nucleons is done by performing a multipole expansion on Eq. (17). At leading order this amounts to evaluating the pion field in Eq. (17) at the space time point $(t, \mathbf{x}) = (t, \mathbf{0})$. Hence, for radiation pions the term in the action corresponding to Eq. (17) is

$$S_{int} = -\frac{g_A}{\sqrt{2}f} \int dt (\nabla^i \pi^j)|_{\mathbf{x}=0} Q^{ij}, \qquad (21)$$

where Q^{ij} are the charges of Wigner symmetry in Eq. (14). In the limit $a^{(s)} \to \infty$ these charges are conserved and the Q^{ij}'s are time independent. Hence, as $a^{(s)} \to \infty$ only the $k^0 = 0$ mode of the pion couples in Eq. (21). This is incompatible with the radiation pion condition, $k^0 \sim \sqrt{\mathbf{k}^2 + m_\pi^2}$. Hence the leading contribution from radiation pions is suppressed by $1/a^{(^1S_0)} - 1/a^{(^3S_1)}$. In a recent paper [17] Mehen and Stewart found this suppression by an explicit calculation of the leading radiation pion contribution to the NN scattering

amplitudes, $\mathcal{A}^{(s)}$. It involved a cancellation between different Feynman diagrams.

6 Outlook

Effective field theory methods are a viable model independent approach to the physics of the two nucleon sector. The power counting is slightly unusual due to the large S-wave NN scattering lengths. This approach is useful up to a center of mass momentum around 200 MeV, however, the expansion parameter at such a momentum is probably not much smaller than $\frac{1}{2}$. It seems likely that for many quantities calculations at N^2LO will reach the same precision as conventional potential model approaches, however, with such a large expansion parameter there are likely to be some failures.

Extension of the effective field theory approach to the three nucleon sector is underway. Several theoretical issues remain to be resolved before there is a complete power counting, but recent progress in this area is very encouraging.

The holy grail of this field is the application of these effective field theory methods to nuclear matter. We are still a long way from having the theoretical tools to tackle this problem and even with these tools the Fermi momentum associated with nuclear density may be too large for a Q expansion to be useful. However, given the importance of understanding the properties of nuclear matter continuing to develop the effective field theory approach is very worthwhile.

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